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ADAPTIVE ROBUST CONTROL SCHEME APPLIED TO A SINGLE-ZONE HVAC SYSTEM

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ABSTRACT

The present paper addresses the modelling and control of a heating, ventilating, and air-conditioning (HVAC) system while the system is operating under uncertainty. The uncertainty may be due to thermal storage effect, heat and moisture generation, and outside temperature and humidity change. These may be unpredictable for a generic room. The adaptive robust control proposed for this system will take the uncertainty into account. No statistical information of the uncertainty is ever needed. The uncertainty is assumed *bounded* but the bound is *unknown*.

1. Introduction

Heating, ventilating, and air-conditioning (HVAC) systems are comprised of a large number of subsystems, each of which may exhibit time-varying and/or nonlinear characteristics. For example, a detailed description of the dynamics of a typical five-zone commercial HVAC system requires on the order of 1,000 differential and algebraic equations (Kelly *et al.* 1984). Furthermore, the parameters of this dynamical description generally vary with load, weather, and building occupancy. These complexities suggest that the use of some simple control schemes (such as on-off control which many HVAC system are using) may not be appropriate for some of the new load-management technologies and systems. This paper endeavors to consider other control alternatives for application to HVAC systems.

Modelling of HVAC component operation using simple temperature control schemes has been studied by several authors. A simple room model having a single-input single-output transfer function was investigated by Zermuehlen (1965) and Harrison *et al.* (1968). Major assumptions include perfect mixing, Newtonian heating, and thermal storage only by the room air. Results were merely illustrative. A duct model was given by Tobias (1973), whose work suggested a way of transforming the governing partial differential equations into ordinary differential equations. Later, Grot and Harrje (1981) presented a similar model, but further assumed force convection as the dominant heat transfer mechanism and neglected storage effects as well as axial wall conduction. An air supply system or discharge air temperature control system (DATCS) was studied by Hamilton *et al.* (1974), and Brandt and Shavit (1984). Hamilton *et al.* treated a DATCS with an air-water heat exchanger as the active control element. Brandt and Shavit simulated the response of a PID-controlled DATCS to a step change input. Similar DATCS systems were the subject of Kurz *et al.* (1980) and Clarke and Gawthrop (1981). This previous work was within the framework of classical control. Fan *et al.* (1970) was probably among the first to introduce modern control concept into the HVAC field. It considered a single room model and discussed the feasibility of modern control applications. Moreover, detailed sensitivity analysis was also investigated. Nakanishi *et al.* (1973) considered the problem of simultaneous temperature and humidity controls. Nonlinear differential equations resulting from the material and mass balances were linearised around two set points corresponding to summer and winter operation. Nakanishi's work demonstrated that modern control formulations could eliminate some of the empiricism used in the classical control design of HVAC control systems. Recent work by Clark *et al.* (1985) involved detailed control models with time delay for ducts, hot water coils, and other air-handling components.

As contrasted to the *component models* aspect discussed in the last paragraph. There is also effort devoted to the *system model* aspect. A system model includes a complete set of HVAC components. Stoecker (1976) modeled an HVAC system with polynomial expressions whose coefficients were determined through experimental or on-site performance data. This formulation is quite valuable for estimating the steady-state operation of an HVAC system. A dynamical model was developed by Thompson and Chen (1979) which included transfer function expressions for various HVAC components. These components were strung together to model an HVAC system. Thompson (1981) later modified the thermostat module. Though these authors developed a digital simulation scheme to identify energy sensitive parameters, they never studied the effects of system dynamics. Mehta (1984) described the concept of a rational model, which includes the dynamic interaction between the HVAC system and the heating/cooling loads. This approach had been suggested by an earlier successful experimental validation (Mehta and Woods 1980) of HVAC models, obtained by linking proper modular blocks. Kaya (1976, 1979, 1981) and Kaya *et al.* (1982) tackled the problem of the optimal control formulation. Temperature, humidity, and air velocity were considered as three major comfort variables, and the comfort condition was treated as a region. A two-step optimisation procedure (static and dynamic) was discussed. Sud (1984) discussed a three-step optimization procedure which included operational modes and control hierarchies. Schumann (1980) presented a simple air-conditioning system using a parameter-adaptive deadbeat controller and a parameter-adaptive optimal state controller. There were also works devoted to the parameter estimation issue which was considered as a major step toward the use of adaptive control. Diderrich and Kelly (1984) described the use of Kalman filtering methods for the failure detection of HVAC sensors. Forrester and Wepfer (1984) and Li and Wepfer (1985) applied off-line least square estimation schemes to data taken from a large commercial office building and developed load prediction algorithm. Later Li and Wepfer (1987) also developed an on-line recursive estimation methods for a multi-input multi-output HVAC system.

The purpose of the present paper is to consider the control issue for an HVAC system which possesses modelling uncertainty and nonlinearity. The uncertainty may be due to thermal storage effect, heat and moisture generation, and outside temperature and humidity. These may be unpredictable for a generic room. The adaptive robust control proposed for this system will compensate the uncertainty. No statistical information of the uncertainty is ever assumed. The uncertainty is assumed *bounded* but the bound is *unknown*. In order to simplify the formulation and hence to emphasize the feature of the control algorithm, only a single-zone HVAC system is considered.

2. A single-zone HVAC system model

A single zone HVAC system in a generic room is considered. The relevant parameters of this prototype are summarised in Tables 1 and 2. The system is constructed by direct application of conservation principles. Assumptions adopted here for the modelling include ideal gas behavior, perfect mixing, negligible radiative heat transfer, and constant pressure.

Conservation of energy leads to (Li and Wepfer 1987):

$$\begin{pmatrix}\text{energy}\\\text{stored}\\\text{in room}\end{pmatrix} = \begin{pmatrix}\text{energy in}\\\text{via air}\\\text{supply}\end{pmatrix} + \begin{pmatrix}\text{heat con-}\\\text{duction}\\\text{via walls}\end{pmatrix} + \begin{pmatrix}\text{heat in}\\\text{due to}\\\text{occupants}\end{pmatrix} - \begin{pmatrix}\text{energy}\\\text{loss via}\\\text{return air}\end{pmatrix} \tag{2.1}$$

Converting the above relationships to symbols (with some terms combined) yields:

$$\frac{dT}{dt} = -\frac{h_{fg} + c_{pw}T}{(V\rho)_{rm}c_p}\left[(F\rho)_e(W_e - W) + \dot{m}_0\right] + \frac{(F\rho)_e(h_e - h)}{(V\rho)_{rm}c_p} + \frac{UA(T_0 - T)}{(V\rho)_{rm}c_p} + \frac{\dot{Q}_0}{(V\rho)_{rm}c_p} \tag{2.2}$$

where all mass-specific quantities are given per kilogram of dry air. Conservation of mass of moisture in the air leads to:

$$\begin{pmatrix}\text{moisture}\\ \text{increase}\\ \text{in air}\end{pmatrix} = \begin{pmatrix}\text{moisture}\\ \text{in via}\\ \text{supply air}\end{pmatrix} + \begin{pmatrix}\text{moisture}\\ \text{in via}\\ \text{occupants}\end{pmatrix} - \begin{pmatrix}\text{moisture}\\ \text{out via}\\ \text{return air}\end{pmatrix} \quad (2.3)$$

In terms of symbols this becomes:

$$\frac{dW}{dt} = \frac{(F\rho)_e}{(V\rho)_{rm}}(W_e - W) + \frac{\dot{m}_0}{(V\rho)_{rm}} \quad (2.4)$$

The following two relationships hold:

$$c_p = \sum_i c_{pi} \quad (2.5)$$

$$\rho = \frac{P}{R(T+273)}\frac{0.622}{(0.622+W)} \sim \text{constant (humid air density)} \quad (2.6)$$

Here in (2.5) the heat capacity is the summation of that of air, wall, furniture, equipment, etc.

According to the ASHRAE standards (ASHRAE 1981), the comfort region (in terms of temperature and humidity) may be approximated as shown in Figure 1. Based on this, we now define the following state variables:

$$x_1 = T^*, \qquad T^* = \frac{T - T_{med}}{T_{max} - T_{min}} \quad (2.7)$$

$$x_2 = W^*, \qquad W^* = \frac{W - W_{med}}{W_{max} - W_{min}} \quad (2.8)$$

$$u_1 = T_e^*, \qquad T_e^* = \frac{T_e - T_{med}}{T_{max} - T_{min}} \quad (2.9)$$

$$u_2 = W_e^*, \qquad W_e^* = \frac{W_e - W_{med}}{W_{max} - W_{min}} \quad (2.10)$$

where "max" and "min" refer to the upper and lower boundaries of the comfort region as shown in Figure 1, $T_{med} = \frac{1}{2}(T_{max}+T_{min})$, and $W_{med} = \frac{1}{2}(W_{max} + W_{min})$. Then (2.2) and (2.4) can be converted to the following state space form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-(\alpha_1+1) & \gamma\Delta W\frac{T_{med}}{\Delta T} + \alpha_2\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1 & -\alpha_2 - \gamma\Delta W(x_1 + \frac{T_{med}}{\Delta T})\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$
$$+ \begin{bmatrix}\alpha_1 & 1 & -\alpha_2 - \gamma\Delta W(x_1 + \frac{T_{med}}{\Delta T})\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u_3\\ u_4\\ u_5\end{bmatrix} + \begin{bmatrix}\gamma\Delta W x_1 x_2\\ 0\end{bmatrix} \quad (2.11)$$

where $\dot{x}_1 = \frac{dx_1}{dt^*}$, $\dot{x}_2 = \frac{dx_2}{dt^*}$, $t^* = tF_e/V_{rm}$, $\Delta T = T_{max} - T_{min}$, $\Delta W = W_{max} - W_{min}$,

$$\alpha_1 = \frac{UA}{\rho c_p F_e}, \qquad \alpha_2 = \frac{h_{fg}\Delta W}{c_p \Delta T}, \qquad \gamma = \frac{c_{pw}}{c_p},$$

$$u_3 = \frac{T_0 - T_{med}}{\Delta T}, \qquad u_4 = \frac{\dot{Q}_0}{c_p F_e \Delta T}, \qquad u_5 = \frac{\dot{m}_0}{F_e \rho \Delta W}.$$

This is a nonlinear system. Moreover, the control u_2 is coupled with the state x_1. In practice for a generic room the thermal storage effects (which may be due to wall, equipment, and furniture, etc.) should not be ignored. However, these are difficult to model in a very precise way. Hence it is realistic to face the fact that the model (2.11) possesses certain degree of uncertainty. We shall treat the heat capacity c_p as an uncertain parameter. To be more specific, let

$$c_p = c_p^{nominal} + \Delta c_p(t) \quad (2.12)$$

where the nominal value of c_p is a known constant. However, the uncertain portion $\Delta c_p(t)$ is time-varying and unpredictable. It is reasonable to assume that $\Delta c_p(t)$ is *bounded*. The bound is however *unknown*. In addition to c_p, there are other uncertainties in the model. These include u_3, u_4, and u_5. Here u_3 is determined by the outside temperature T_0 which may be varying due to weather change. The other two inputs u_4 and u_5 are related to the heat and moisture generation (through $\dot{Q}_0$ and $\dot{m}_0$). We shall also treat u_3, u_4, and u_5 as uncertainties. These are time-varying and their changes are unpredictable. However, it is again realistic to assume that the variations (around certain nominal values) are *bound*. The bound is however *unknown*.

3. Adaptive robust control

The dynamic model of the single zone HVAC system described in the last section possesses modelling uncertainty. It is then desirable to design control which is able to take the uncertainty into account. The control purpose is to drive the room's state (which is related to its temperature and humidity) into the comfort region (as shown in Figure 1). In this section, a class of adaptive robust controls which was originally designed by Corless and Leitmann (1984) is first introduced. The merit of the control is that it is able to compensate the uncertainty without knowing what the uncertainty is.

Consider the following class of uncertain systems:

$$\dot{x}(t) = Ax(t) + \Delta f(x(t),t) + [B(x(t),t) + \Delta B(x(t),t)]u(t), \quad (3.1)$$

where $t \in \mathbb{R}$, $x(t) \in \mathbb{R}^n$ is the state, and $u(t) \in \mathbb{R}^m$ is the control. The matrices A, $\Delta f(x,t)$, $B(x,t)$, and $\Delta B(x,t)$ are of appropriate dimensions. The matrix A and the function $B(\cdot)$ are both *known*. The functions $\Delta f(\cdot)$ and $\Delta B(\cdot)$ are *uncertain*. That is, they are not assumed known but are assumed to satisfy certain conditions (to be stated in Assumption (2)). We now state the following assumptions.

Assumption (1): The matrix A is Hurwitz.

Assumption (2): (1) The functions $\Delta f(\cdot)$ and $\Delta B(\cdot)$ are continuous.

(2) There exist *uncertain* functions $h(\cdot)$ and $E(\cdot)$ such that for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}$,

$$\Delta f(x,t) = B(x,t)h(x,t), \quad (3.2)$$

$$\Delta B(x,t) = B(x,t)E(x,t). \quad (3.3)$$

(3) There exists a (unknown) constant λ such that for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}$,

$$\min \lambda_m \frac{1}{2}[E^T(x,t) + E(x,t)] \geq \lambda > -1. \quad (3.4)$$

Here $\lambda_m(\cdot)$ ($\lambda_M(\cdot)$) is the minimum (maximum) eigenvalue of the designated matrix.

(4) There exist an *unknown* constant vector $\beta \in (0,\infty)^q$ and a *known* continuous function $\rho(\cdot): \mathbb{R}^n \times \mathbb{R} \times (0,\infty)^q$ such that for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}$,

$$\|h(x,t)\| \leq \rho(x,t,\beta). \quad (3.5)$$

Throughout this article vector norm is taken to be Euclidean and matrix norm is the corresponding induced one; thus for a matrix Υ, $\|\Upsilon\|^2 = \lambda_M(\Upsilon^T\Upsilon)$.

(5) For each $(x,t) \in \mathbb{R}^n \times \mathbb{R}$, the function $\rho(x,t,\cdot): (0,\infty)^q \to \mathbb{R}_+$ is C^1, concave, and non-decreasing with respect to each coordinate of its argument, β. Here the *concave* property means that for any β^1, $\beta^2 \in (0,\infty)^q$,

$$\rho(x,t,\beta^1) - \rho(x,t,\beta^2) \leq \frac{\partial\rho}{\partial\beta}(x,t,\beta^2)(\beta^1 - \beta^2). \quad (3.6)$$

Remark (1): Assumption (1) imposes condition on the *nominal* portion of the system (3.1). Notice that for a given system, the way of choosing A and $\Delta f(x,t)$, etc. is not unique. This thus suggests the way of decomposing the uncertain system (3.1) (that is, one first "chooses" a matrix A and then lumps the rest of the system portion in $\Delta f(x,t)$).

Remark (2): The condition on the constant λ assures that the adaptive robust control proposed later for the system (3.1) can "act" in the desired direction. That is, the control direction is not deteriorated by the uncertain portion $E(x,t)$. However, it is possible that the control magnitude is affected by the uncertain portion $E(x,t)$. The positive constant vector β can be interpreted as is related to the bound of $\Delta f(x,t)$ and $\Delta B(x,t)$. However, the relationship may not be direct. Notice that the dimension of β (i.e., q) can be different from that of $\Delta f(x,t)$, etc. That β is unknown reflects that the bound is unknown. However, certain properties (shown in Assumption 2(5)) on how the system depends on β are *known*.

We now propose the following class of adaptive robust controls:

$$u(t) = p(x(t), t, \hat{\beta}(t), \epsilon(t)), \tag{3.7}$$

$$p(x,t,\hat{\beta},\epsilon) = -\frac{\mu(x,t,\hat{\beta})}{\|\mu(x,t,\hat{\beta})\|}\rho(x,t,\hat{\beta}) \quad \text{if } \|\mu(x,t,\hat{\beta})\| > \epsilon, \tag{3.8.1}$$

$$p(x,t,\hat{\beta},\epsilon) = -\frac{\mu(x,t,\hat{\beta})}{\epsilon}\rho(x,t,\hat{\beta}) \quad \text{if } \|\mu(x,t,\hat{\beta})\| \le \epsilon, \tag{3.8.2}$$

$$\dot{\hat{\beta}}(t) = L\frac{\partial\rho^T}{\partial\beta}(x(t),t,\hat{\beta}(t))\|\nu(x(t),t)\|, \tag{3.9}$$

$$\dot{\epsilon}(t) = -l\epsilon(t), \tag{3.10}$$

$$\hat{\beta}(t_0) \in (0,\infty)^q, \qquad \epsilon(t_0) \in (0,\infty), \tag{3.11}$$

where $\nu(x,t) = B(x,t)^T Px$, $P > 0$ is the solution of the Lyapunov equation $A^T P + PA + Q = 0$, $Q > 0$, $\mu(x,t,\hat{\beta}) = \nu(x,t)\rho(x,t,\hat{\beta})$, $L \in \mathbf{R}^{q\times q}$ is diagonal with positive elements, and $l > 0$.

The controlled system and the adaptive scheme can then be expressed as follows:

$$\begin{aligned}\dot{x}(t) =& Ax(t) + \Delta f(x(t),t)\\ &+ [B(x(t),t) + \Delta B(x(t),t)]p(x(t),t,\hat{\beta}(t),\epsilon(t)),\end{aligned} \tag{3.12}$$

$$\dot{\hat{\beta}}(t) = L\frac{\partial\rho^T}{\partial\beta}(x(t),t,\hat{\beta}(t))\|\nu(x(t),t)\|, \tag{3.13}$$

$$\dot{\epsilon}(t) = -l\epsilon(t). \tag{3.14}$$

The resulting controlled system performance is described as follows. We first define the parameter "estimate"† vector

$$\hat{\xi}(t) = (\hat{\beta}(t)^T \; \epsilon(t))^T, \tag{3.15}$$

$$\hat{\xi}(t_0) = (\hat{\beta}(t_0)^T \; \epsilon(t_0))^T \in (0,\infty)^{q+1}, \tag{3.16}$$

and the parameter vector

$$\xi = (\beta^T \; 0)^T. \tag{3.17}$$

Theorem (1): Consider the dynamic system described by (3.1). Suppose that Assumptions (1) and (2) are met. As the adaptive robust control (3.8-10) is applied to (3.1), the resulting controlled system and the adaptive scheme can be described by (3.12-14) and has the following properties.

(i) *Existence of solutions.* For each $(x_0, t_0, \hat{\xi}(t_0)) \in \mathbf{R}^n \times \mathbf{R} \times (0,\infty)^{q+1}$ there exists a solution $(x(\cdot), \hat{\xi}(\cdot)) : [t_0, t_1) \to \mathbf{R}^n \times (0,\infty)^{q+1}$ of (3.12-14) with $(x(t_0), \hat{\xi}(t_0)) = (x_0, \hat{\xi}_0)$.

(ii) *Uniform stability of* $(0, \xi)$. For each $\eta > 0$ there exists $\varsigma > 0$ such that if $(x(\cdot), \hat{\xi}(\cdot))$ is any solution of (3.12-14) with $\|x(t_0)\|$, $\|\hat{\xi}(t_0) - \xi\| < \varsigma$ then $\|x(t)\|$, $\|\hat{\xi}(t) - \xi\| < \eta$ for all $t \in [t_0, t_1)$.

(iii) *Uniform boundedness of solutions.* For each r_1, $r_2 > 0$ there exist $d_1(r_1,r_2)$, $d_2(r_1,r_2) \ge 0$ such that if $(x(\cdot), \hat{\xi}(\cdot))$ is any solution of (3.12-14) with $\|x(t_0)\| \le r_1$ and $\|\hat{\xi}(t_0) - \xi\| \le r_2$ then $\|x(t)\| \le d_1(r_1,r_2)$ and $\|\hat{\xi}(t) - \xi\| \le d_2(r_1,r_2)$ for all $t \in [t_0, t_1)$.

(iv) *Extension of solutions.* Every solution of (3.12-14) can be extended into a solution defined on $[t_0, \infty)$.

(v) *Convergence of* $x(\cdot)$ *to zero.* If $(x(\cdot), \hat{\xi}(\cdot)) : [t_0, \infty) \to \mathbf{R}^n \times (0,\infty)^{q+1}$ is a solution of (3.12-14) then

$$\lim_{t\to\infty} x(t) = 0. \tag{3.18}$$

Proof: See Corless and Leitmann (1984).

Remark (3): The adaptive robust control is of saturation type. The direction of the control is prespecified to be $-\mu$. The magnitude of the control is however determined by whether the state variable x is outside of the saturation region $\|\mu\| = \epsilon$ (hence (3.8.1)) or inside (hence (3.8.2)). The control is designed without knowing the uncertainty. This fits the requirement of the single zone HVAC control problem. It is *guaranteed* that the state $x(t)$ converges to zero.

Remark (4): There are certain design parameters involved. The value of L determines the rate of adaptation. A larger value of L implies a faster learning rate. It can be shown that if the initial condition $\hat{\xi}(t_0)$ is chosen positive (which is required in this adaptive scheme) then the parameter "estimate" vector $\hat{\xi}(t)$ remains positive for all $t \in [t_0, \infty)$ (Corless and Leitmann 1984). This fits the physical interpretation of $\hat{\beta}$ (which is a part of $\hat{\xi}$) that it is related to the bound of the uncertainty. The parameter $\epsilon(t)$ is governed by (3.10) which is decoupled with the state variable $x(t)$ and the adaptive parameter $\hat{\beta}(t)$. The value of ϵ determines the size of the saturation region $\|\mu\| = \epsilon$ for the adaptive robust control (3.7). In fact (3.10) shows that $\epsilon(t)$ converges to zero asymptotically. The choice of the initial condition $\epsilon(t_0)$ and the constant l is arbitrary (as long as they are positive). Hence one can manipulate the value of ϵ (at least initially) and hence the size of the saturation region in practical application.

4. Implementation of the Adaptive Robust Control

We now apply the adaptive robust control to the single zone HVAC system. To simplify the problem, we shall assume that room temperature and humidity measurement are both available. We first decompose the system (2.11) into the nominal and uncertain portions. Based on the decomposition of c_p performed in (2.12), we are able to decompose α_1, α_2, and γ such that

$$\alpha_1 = \alpha_1^{nominal} + \Delta\alpha_1(t), \tag{4.1}$$

$$\alpha_1^{nominal} = \frac{UA}{\rho c_p^{nominal} F_e},$$

$$\Delta\alpha_1(t) = \frac{UA}{\rho F_e}\frac{-\Delta c_p(t)}{(c_p^{nominal} + \Delta c_p(t))c_p^{nominal}},$$

$$\alpha_2 = \alpha_2^{nominal} + \Delta\alpha_2(t), \tag{4.2}$$

$$\alpha_2^{nominal} = \frac{h_{fg}\Delta W}{\Delta T c_p^{nominal}},$$

$$\Delta\alpha_2(t) = \frac{h_{fg}\Delta W}{\Delta T}\frac{-\Delta c_p(t)}{(c_p^{nominal} + \Delta c_p(t))c_p^{nominal}},$$

$$\gamma = \gamma^{nominal} + \Delta\gamma(t), \tag{4.3}$$

$$\gamma^{nominal} = \frac{c_{pw}\Delta W}{c_p^{nominal}}, \qquad \Delta\gamma(t) = \frac{c_{pw}\Delta W(-\Delta c_p(t))}{(c_p^{nominal} + \Delta c_p(t))c_p^{nominal}}.$$

Notice again that $\Delta c_p(t)$ is *unknown*. No deterministic or statistical information is assumed. Comparing (2.11) with (3.1), we choose

$$A = \begin{bmatrix} -(\alpha_1^{nominal} + 1) & \gamma^{nominal}\Delta W\frac{T_{set}}{\Delta T} + \alpha_2 \\ 0 & -1 \end{bmatrix}, \tag{4.4}$$

$$\Delta f(x,t) = \begin{bmatrix} -\Delta\alpha_1(t) & \Delta\gamma(t)\Delta W\frac{T_{set}}{\Delta T} + \Delta\alpha_2(t) \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_1 & 1 & -\alpha_2 - \gamma\Delta W(x_1 + \frac{T_{set}}{\Delta T}) \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_3 \\ u_4 \\ u_5 \end{bmatrix} + \begin{bmatrix} \gamma\Delta W x_1 x_2 \\ 0 \end{bmatrix}, \tag{4.5}$$

$$B(x,t) = \begin{bmatrix} 1 & -\alpha_2^{nominal} - \gamma^{nominal}\Delta W(x_1 + \frac{T_{set}}{\Delta T}) \\ 0 & 1 \end{bmatrix}, \tag{4.6}$$

$$\Delta B(x,t) = \begin{bmatrix} 0 & -\Delta\alpha_2(t) - \Delta\gamma(t)\Delta W(x_1 + \frac{T_{set}}{\Delta T}) \\ 0 & 0 \end{bmatrix}. \tag{4.7}$$

In order to implement the control, we need to show the satisfactions of Assumptions (1) and (2). The matrix A is Hurwitz. The matching condition (3.2,3.3) is met since the square matrix B in (4.6) is nonsingular (hence one may choose $h = B^{-1}\Delta f$ and $E = B^{-1}\Delta B$). Assumption 2(3) is best verified by simulation result. This is since E depends on x and in practice one only has to satisfy (3.4) for all

† This is not to be interpreted as $\hat{\xi}(t) \to \xi$. All this means is that $\hat{\beta}$ is used in the control scheme (3.7) in place of β.

system solution $x(\cdot)$. Next we need to perform the bounding analysis on $h(x,t)$. It is sufficient to show that $\Delta f(x,t)$ satisfies Assumption 2(4,5) since $\|h(x,t)\| \leq \|B^{-1}\|\|\Delta f(x,t)\|$ and $\|B^{-1}\| = 1$ (although B depends on x_1). We first show that the first two terms on the right-hand side of (4.5) are cone-bounded. That is, there exist (unknown) constants β_2 and β_3 such that

$$\|\Psi_1 x + \Psi_2(x)\bar{u}\| \leq \beta_2\|x\| + \beta_3, \tag{4.8}$$

where

$$\Psi_1 x = \begin{bmatrix} -\Delta\alpha_1(t) & \Delta\gamma(t)\Delta W(\frac{T_{set}}{\Delta T} + \Delta\alpha_2(t)) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{4.9}$$

$$\Psi_2(x) = \begin{bmatrix} \alpha_1 & 1 & -\alpha_2 - \gamma\Delta W(x_1 + \frac{T_{set}}{\Delta T}) \\ 0 & 0 & 1 \end{bmatrix}, \tag{4.10}$$

$$\bar{u} = \begin{bmatrix} u_3 \\ u_4 \\ u_5 \end{bmatrix}.$$

It is clear that

$$\|\Psi_1 x\| \leq \|\Psi_1\|\|x\|. \tag{4.11}$$

We separate the matrix $\Psi_2(x)$ into two parts:

$$\begin{aligned} \Psi_2(x) &= \begin{bmatrix} \alpha_1 & 1 & -\alpha_2 - \gamma\Delta W\frac{T_{set}}{\Delta T} \\ 0 & 0 & 1 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & -\gamma\Delta W \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x_2 \\ 0 & 0 & x_1 \end{bmatrix} \\ &= \Psi_{21} + \Psi_{22} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x_2 \\ 0 & 0 & x_1 \end{bmatrix} \end{aligned} \tag{4.12}$$

Then a simple bounding analysis yields

$$\|\Psi_2(x)\bar{u}\| \leq \max\|\bar{u}\|(\|\Psi_{21}\| + \|\Psi_{22}\|\|x\|) \tag{4.13}$$

The cone-boundedness property (4.8) is proven. In fact, we can choose the constants β_2 and β_3 to be

$$\beta_2 = \|\Psi_1\| + \max\|\bar{u}\|\|\Psi_{22}\| \tag{4.14}$$

$$\beta_3 = \max\|\bar{u}\|\|\Psi_{21}\| \tag{4.15}$$

The third term on the right-hand side of (4.5) is bounded by $\|x\|^2$ since $2|x_1 x_2| \leq x_1^2 + x_2^2 = \|x\|^2$ and hence

$$\|\Psi_3(x)\| \leq \max|\gamma\Delta W|\|x\|^2 \triangleq \beta_1\|x\|^2, \tag{4.16}$$

where

$$\Psi_3(x) = \begin{bmatrix} \gamma\Delta W x_1 x_2 \\ 0 \end{bmatrix}. \tag{4.17}$$

Combining (4.8) and (4.16) we conclude that

$$\|h(x,t)\| \leq \beta_1\|x\|^2 + \beta_2\|x\| + \beta_3 \triangleq \rho(x,t,\beta) \tag{4.18}$$

It is then easy to check that Assumption 2(4,5) is met. The adaptive scheme is now constructed as follows:

$$\dot{\hat{\beta}}_1(t) = L_1\|\nu(x)\|\|x\|^2 \tag{4.19}$$

$$\dot{\hat{\beta}}_2(t) = L_2\|\nu(x)\|\|x\| \tag{4.20}$$

$$\dot{\hat{\beta}}_3(t) = L_3\|\nu(x)\| \tag{4.21}$$

where L_1, L_2, and L_3 are the diagonal elements of the matrix L. The adaptive robust control for the single zone HVAC system is given by (3.7) with

$$\rho(x,t,\beta) = \beta_1\|x\|^2 + \beta_2\|x\| + \beta_3 \tag{4.22}$$

where $\beta = [\beta_1\ \beta_2\ \beta_3]^T$.

Computer simulations are performed for system analysis. The parameters for the single zone prototype are summarized in Tables 1 and 2. The uncertain parameters (i.e., $\dot{Q}_0(t)$, $\dot{m}_0(t)$, $T_0(t)$, and $c_p(t)$) are decomposed with their nominal values given in Table 1. Their uncertain portions are given in the following form for simulation purposes:

$$\Delta\dot{Q}_0(t) = a_1 + b_1 \sin\left[\frac{2\pi t}{c_1}\right] + d_1\, norm(t) \tag{4.23}$$

$$\Delta\dot{m}_0(t) = a_2 + b_2 \sin\left[\frac{2\pi t}{c_2}\right] + d_2\, norm(t) \tag{4.24}$$

$$\Delta T_0(t) = a_3 + b_3 \sin\left[\frac{2\pi t}{c_3}\right] + d_3\, norm(t) \tag{4.25}$$

$$\Delta c_p(t) = a_4 + b_4 \sin\left[\frac{2\pi t}{c_4}\right] + d_4\,[rect(t) - 0.5] \tag{4.26}$$

where $norm(t)$ is a random number with mean=0 and standard deviation=1 and $rect(t)$ is a random number with rectangular distribution in the interval [0, 1]. The purpose of using these forms of functions for the uncertainty in simulations is to consider the combinations of various practical situations, including constant uncertainty, high frequency periodic uncertainty, and random uncertainty. The numerical values of the parameters a_1, etc. which are adopted for simulations are summarized in Table 2. Moreover, we take the outside temperature $T_0 = 35°C$, the heat gain $\dot{Q}_0 = 75\ W$ hourly, and the moisture load $\dot{m}_0 = 3 \times 10^{-5}\ kg/s$ hourly.

For comparison purpose, an on-off control (which many HVAC systems are using) is also implemented for the single zone system under the same uncertainty. The control is given in the following form:

$$u_1(t) = \begin{Bmatrix} -u_{1max} \text{ if } x_1 > \delta_1 \\ 0 \text{ if } -\delta_1 \leq x_1 \leq \delta_1 \\ u_{1max} \text{ if } x_1 < -\delta_1 \end{Bmatrix}, \tag{4.27}$$

$$u_2(t) = \begin{Bmatrix} -u_{2max} \text{ if } x_2 > \delta_2 \\ 0 \text{ if } -\delta_2 \leq x_2 \leq \delta_2 \\ u_{2max} \text{ if } x_2 < -\delta_2 \end{Bmatrix}, \tag{4.28}$$

where u_{imax}, $i = 1,2$, is the maximum control magnitude, $[-\delta_i,\ \delta_i]$ is the dead-zone (in terms of the control action). For simulation purpose, we take $u_{1max} = 500$, $u_{2max} = 300$, $\delta_1 = 0.5$, and $\delta_2 = 0.5$. The other parameters chosen for the adaptive robust control (3.7) are: $L_1 = L_2 = L_3 = 1$, $l = 0.01$, $\epsilon(0) = 10$, $\hat{\beta}_1(0) = \hat{\beta}_2(0) = \hat{\beta}_3(0) = 10$.

The initial conditions of the system are chosen to be $x_1(0) = 6$ and $x_2(0) = 4$. This significant deviation from the comfort region is intended to test the recovery capability of the control system in a severe situation. Figures 2-7 depict the system and control performances. The time axis is scaled such that it is equivalent to $100tF_e/V_{rm}$. In other words, a unit time in the figure is equivalent to 28.60 seconds based on the prototype parameters in Table 1. Figure 2 is the state performance under no control. Figure 3 is due to the use of the on-off control. Figures 4-7 are due the use of the adaptive robust control. It is interesting to note that the system performance due to the control (4.27,28) has a very high temperature overshoot (shown as curve 1 in Figure 3). This is mainly due to the coupling of the control u_2 with x_1 (as shown in (2.11)). However, the steady state performance is rather satisfactory. On the other hand, the system performance due to the adaptive robust control shows a much less overshoot (Figure 6). The steady state performance has certain oscillations. Due to the practical need for room comfort, it is usually more important to be able to maintain small overshoot than to have a slight improvement in the steady state performance. A human body can not always tell the difference of 1°C (which is about the difference between the steady state values of Figures 3 and 4). However, the overshoot in Figure 3 certainly indicates a significant discomfort. This comparison in fact also suggests the practical need for a realistic room temperature-humidity control system. It is more important that the control system is *robust* against the uncertainty (in the sense of maintaining small overshoot) than showing a slight improvement in the steady state performance.

5. Conclusions

We have developed a class of adaptive robust controls for a single-zone HVAC system. The system possesses modelling uncertainty and nonlinearity. The uncertainty under consideration includes thermal storage effect, heat and moisture generation, and outside temperature and humidity variation. The adaptive robust control was designed based on the nominal portion of the uncertain system as well as certain functional properties of the uncertainty bound (the bound itself is however unknown). Simulation results depict a satisfactory transient performance under a significant deviation of the initial state from the comfort region. It is suggested that a realistic and reliable HVAC control system should be able to be *robust* (in the sense that a reasonably small overshoot can be maintained) against uncertainty.

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Nomemclature

A	cross-sectional area (m^2)
e	subscript, supply air
F	volumetric flow rate (m^3/s)
h_{fg}	latent heat of water (J/kg)
h	enthalpy ($J/kg - dry\ air$)
$\dot{m}_0$	internal moisture load (kg/s)
P	pressure (Pa)
$\dot{Q}_0$	internal heat load (W)
R	specific gas constant (J/kg^oK)
rm	subscript, room
t	time (s)
T	temperature (^oC)
U	heat transfer coefficient ($W/m^{2o}K$)
V	volume (m^3)
W	moisture/dry air ratio by mass
ρ	air density

Type	Room	Work Intensity	Light
U value, wall ($W/m^2\ K$)	1.42	V_{rm} (m^3)	27
U value, window ($W/m^2\ K$)	6.42	F_e (m^3/s)	9.44E-03
UA value (W/K)	22.78	c_{pa} ($J/kg\ K$)	1005
h_{fg} (J/kg)	2.501E+06	c_{pw} ($J/kg\ K$)	1820
P (MPa)	0.101	R_{air} ($J/kg\ K$)	287
ΔT (K)	4	ρ_{air} (kg/m^3)	1.2
ΔW (kg_w/kg_a)	0.0076	$100t^*$ Time scale (s)	28.60

Table 1: Descriptive Parameters for the Single-zone Prototype

	$i = 1$	2	3	4
a	3	1×10^{-5}	2	300
b	10	1×10^{-5}	8	300
c	0.01	1	1	1
d	1	1×10^{-5}	1	30

Table 2: Parameters Used for Uncertainty

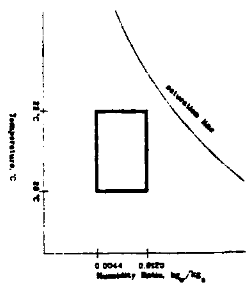

Figure 1: The comfort zone

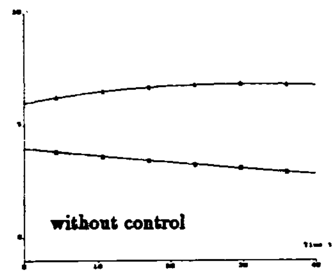

Figure 2: System performance (x_1 and x_2)

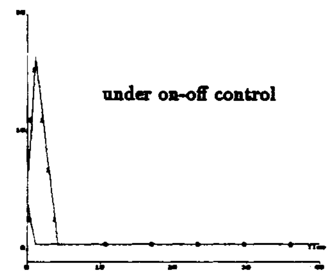

Figure 3: System performance (x_1 and x_2)

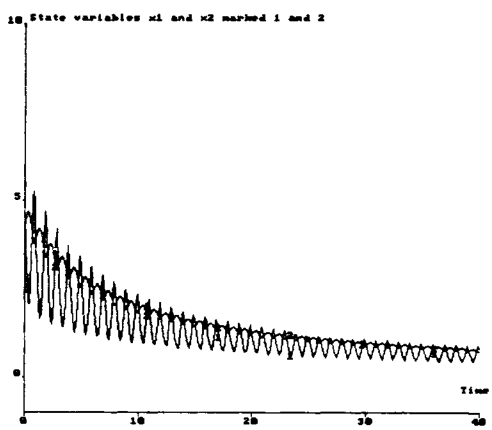

Figure 4: System performance (x_1 and x_2), under adaptive robust control

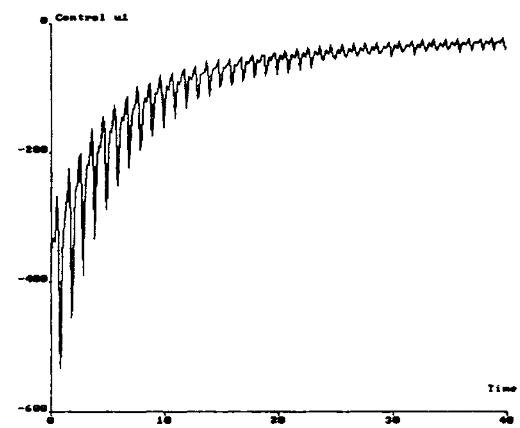

Figure 5: Control history (u_1), adaptive robust control

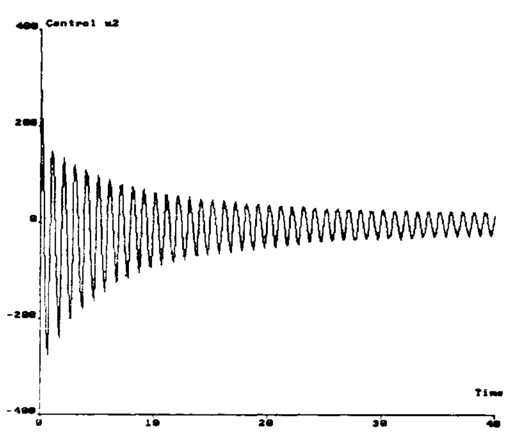

Figure 6: Control history (u_2), adaptive robust control

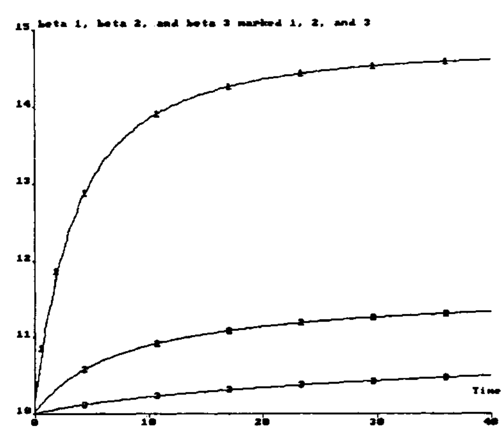

Figure 7: Adaptive parameters history ($\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$)