Equivalent Voice-coil Models for Real-time Computation in Electromagnetic Actuation and Sensor Applications

Kok-Meng Lee*, Fellow, IEEE and Hungsun Son

Abstract—This paper presents a method to derive equivalent models to characterize the magnetic field and force of a multi-layer (ML) voice coil. Two equivalent models are discussed. The first reduces the number of wire layers to a minimum. To offer some intuitive insights, a detailed derivation of an equivalent single layer (ESL) model is given in this paper. The second models the ML coil as an equivalent PM, the magnetic field of which has a closed-form solution. The equivalent models are validated by investigating the effects of coil geometry on the modeling errors, and by comparing the computed forces against published data. As illustrated with a number of examples in this paper, the field and force calculations which do not increase with the number of current loop-turns offer a number of advantages in real-time applications.

Index Terms—Electromagnetic field modeling, Lorentz force, actuator design, spherical motor

I. INTRODUCTION

Iron-less voice coils are commonly used in mechatronic devices (such as hard-disk drives and multi-DOF actuators) due to their linearity as they are free from iron saturation, and the wide availability of high-coercive rare-earth permanent magnets at low cost. The ability to calculate the magnetic fields and forces in real-time can offer a number of advantages. These include accounting for the effects of the self and mutual inductances of the voice coils and the back electromotive forces (emf) on the voltage controlled devices, as well as offering effective motion estimation for model-based control of electromagnetic actuators.

Magnetic forces exerted on current-carrying conductors in a magnetic field are often calculated by the use of Lorentz force equation, which does not involve the magnetic flux generated by the current loop as the current density vector is directly used in the calculation. However, the three-dimensional (3D) integral of the Lorentz force equation must account for each of the current-carrying conductors. For devices such as a spherical motor [1]-[5] where a large number of coils (with multi layers of wires in each) are used, the field and force calculations are often very time consuming for real-time applications. A common approach to accurately compute the magnetic fields has been the use of numerical methods such as FEM. Numerical methods such as FEM offer a good prediction of the magnetic field for accurate computation of the magnetic torque. However, demanding computational time limits these numerical methods to off-line computation.

In [6], we introduce an alternative method, referred here as distributed multi-pole (DMP) modeling, to compute the magnetic fields of permanent magnets using a distributed set of dipoles enabling the magnetic flux density to be computed in closed form. The concept of a magnetic dipole is also commonly used to characterize the magnetic field of a single circular loop carrying current. However, unlike the magnetic dipole [7] derived on the basis of a vector potential for a single current loop, the equivalent models introduced here take into account the physical dimension of a multi-layer coil in modeling the magnetic fields. Yet, as in the case of a magnetic dipole the equivalent PM model of the coil offers the field solutions in closed form. Once the magnetic fields of both the permanent magnets and voice coils are obtained in closed form, they can be computed in real-time for motion estimation.

In this paper, two equivalent models to reduce computational time for calculating the magnetic fields and forces involving multilayer (ML) voice coils are introduced; an equivalent single layer (ESL) model, and an equivalent permanent magnet (PM) model. The first method (or the ESL model) reduces the original ML voice coil to an equivalent model, which retains the shape of the original coil but with only a minimum number of wire layers. The second method models the original ML voice coil, as an equivalent PM which can then be characterized by a distributed set of magnetic dipoles or simply the DMP [6] model. The DMP model inherits many advantages of the dipole model originally conceptualized in the context of physics [8]-[12], but provides an effective means to account for the shape and magnetization of the physical magnet. As will be shown, the field and force calculations do not increase with the number of turns once the equivalent model is found.

The remainder of this paper offers the following:

1) We formulate and derive two equivalent models for efficient computing the magnetic flux density of a voice coil. The key to this method is to find an effective radius and current density (or a magnetization vector for the case of an equivalent PM) such that the equivalent models closely approximates the magnetic flux density.

2) We validate the equivalent model by investigating the effects of coil geometry on the modeling errors, and by comparing the computed forces against published data.

3) We illustrate two applications of the equivalent PM model; a pole-shape design and field-based orientation sensing. Unlike numerical solutions such as finite element methods, the magnetic field solutions obtained using the equivalent PM model are in closed form and thus well suit for real-time computational applications.

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II. EQUIVALENT MODELS OF A MULTI-LAYER VOICE COIL

The design of electromagnetic actuators and sensors involves calculation of magnetic fields and forces due to a current-carrying voice coil. For a thin wire with cross-section area $S$, the magnetic field density caused by the current $I$ flowing along the wire can be determined by the Biot-Savart law:

$$\mathbf{B}_s = \oint d\mathbf{B}_s \quad \text{where} \quad d\mathbf{B}_s = \frac{\mu_0 I}{4\pi} \frac{d\mathbf{s} \times \mathbf{e}_y}{|\mathbf{R} - \mathbf{R}'|} \quad (1)$$

where $d\mathbf{s}$ is an elemental length vector of the wire; $\mathbf{e}_y$ is the unit vector from the source point $\mathbf{R}'$ to the field point $\mathbf{R}$; $I = \oint d\mathbf{s}$; $J$ is the current density; and $\mu_0$ is free space permeability. The Lorentz force exerted on the current-carrying conductor due to the magnetic flux density $\mathbf{B}_s$ can be calculated using (2):

$$\mathbf{F} = -\oint \mathbf{B}_s \times d\mathbf{l} \quad \text{where} \quad I = \oint J \cdot d\mathbf{l} \quad (2)$$

where $d\mathbf{l}$ is the normalized current direction vector.

In (1) and (2), the integral must account for each of the current-carrying conductors. An effective method to reduce computation time is to replace the multilayer (ML) coil with an equivalent single-layer (ESL) model. In general, the ESL model retains the shape of the original ML coil but with only one layer of wires. For example, for a cylindrical coil the process involves finding an effective radius $a_e$ and current density $J_e$. The unknown parameters are chosen such that the errors of the magnetic flux along the centroidal axis are minimized, and that the same magnetic flux density is generated at the end surface of the core. As will be shown, the field and force calculations of the ESL model do not increase with the number of turns.

A. Equivalent Single-layer (ESL) Model

Consider a typical multilayer (axi-symmetrical) coil with a current density $J$, the sectional view of which is shown in Fig. 1(a), where some analytical solutions are available for model validation. However, the method can be extended to coils of other customized shape.

To find the switching radius, we consider the 2D magnetic flux density as shown in Fig. 1(a). For a single wire,

$$\mathbf{B}(y', z') = \frac{\mu_0 I}{2\pi} (1 \times \mathbf{e}_y) \quad \text{where} \quad I = \int d\mathbf{s} \quad (3)$$

The total magnetic flux densities at any point (distance vector $\mathbf{R}$) can be calculated by integrating over the current-carrying conductor. For the original ML coil (with inner and outer radii, $a_i$ and $a_o$, respectively)

$$\mathbf{B}_{ml}(y, z) = \frac{\mu_0 I}{2\pi} \int_{y=0}^{y_2} \int_{r=a_i}^{r=a_o} \frac{1}{|\mathbf{R} - \mathbf{R}'|} d\mathbf{R} \cdot d\mathbf{s} \quad (4a)$$

where

$$d\mathbf{R} = \left[ \begin{array}{c} dx' \\ dy' \\ dz' \end{array} \right] = \left[ \begin{array}{c} dx \\ dy \\ dz \end{array} \right] \left[ \begin{array}{c} \frac{dx}{dr} \cos \theta - \frac{dy}{dr} \sin \theta \\ \frac{dx}{dr} \sin \theta + \frac{dy}{dr} \cos \theta \\ dz \end{array} \right]$$

Similarly, for a single layer coil,

$$\mathbf{B}_{sl}(y, z) = \frac{\mu_0 J_e}{2\pi} \int_{y=0}^{y_2} \int_{r=a_i}^{r=a_o} \frac{1}{|\mathbf{R} - \mathbf{R}'|} d\mathbf{R} \cdot d\mathbf{s} \quad (4b)$$

where

$$|\mathbf{R} - \mathbf{R}'| = \sqrt{(y-y')^2 + (z-z')^2}$$

As the magnetic flux is dominant along the centroidal axis, the unknown parameters ($J_e$ and $a_e$) are determined to satisfy two conditions:

**Condition I:** Minimize the difference between the two models defined by (5):

$$E_y = \int_{y=0}^{y_2} \left[ B_{ml}(y, z) - B_{sl}(y, z) \right] dy \quad (5)$$

where $B_{ml}(y, z) = B_{ml}(y, z)\mathbf{e}_y$ and $B_{sl}(y, z) = B_{sl}(y, z)\mathbf{e}_y$.

Note that $\cos \theta = (y-y')/|\mathbf{R} - \mathbf{R}'|$, we have

$$B_{sl}(y, \ell/2) = \frac{\mu_0 J_e \ell / (2\pi)}{2 \ln \left( \frac{1 + \frac{\ell}{2\ell} - \frac{\ell}{2\ell} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 + \frac{\ell}{2\ell} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 }{ \frac{1}{1 + \frac{\ell}{2\ell}} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 + \frac{\ell}{2\ell} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 } \right.)} \quad (5a)$$

$$J_e d_s = \int_{1/2}^{1} \int_{a_i}^{a_o} \frac{1}{|\mathbf{R} - \mathbf{R}'|} d\mathbf{R} \cdot d\mathbf{s} \quad (6)$$

where $\mathbf{R}' = (a + y) \mathbf{e}_y$, and $\mathbf{R} = (a + y) \mathbf{e}_y$. The unknown parameters $(a_i$ and $J_e$) can be solved simultaneously from (5) and (6). For an axi-symmetrical coil, a 2D model as shown in Fig. 1(a) is sufficient for deriving the unknown parameters of the ESL model. However, the 3D magnetic flux density is needed for field calculation, which can be obtained by applying the Biot-Savart law in (7). For the original ML coil,

$$\mathbf{B}_{ml} = \frac{\mu_0 J_e \ell / (2\pi)}{2 \ln \left( \frac{1 + \frac{\ell}{2\ell} - \frac{\ell}{2\ell} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 + \frac{\ell}{2\ell} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 }{ \frac{1}{1 + \frac{\ell}{2\ell}} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 + \frac{\ell}{2\ell} \left( \frac{1}{1 + \frac{\ell}{2\ell}} \right)^2 } \right.)} \quad (7)$$

where $|\mathbf{R} - \mathbf{R}'| = (x - r \cos \theta)^2 + (y - r \sin \theta)^2 + (z - z')^2$;
\[ e_{\psi} = -\sin \psi e_{x} + \cos \psi e_{z} \text{; and} \]
\[ \psi = \cos^{-1} \left( \sqrt{(x - r \cos \theta)^2 + (y - r \sin \theta)^2}/|\mathbf{R} - \mathbf{R'}| \right). \]

In (7), the negative sign is due to the cross product in the coordinate system of Fig. 1(b). Similarly, once \( a_i \) and \( J_i \) are found the 3D magnetic flux density can be derived from the equivalent single layer (ESL) model:

\[ B_{\text{SL}} = -\mu J_i d_i \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \int_{0}^{2\pi} \frac{1}{|\mathbf{R} - \mathbf{R'}|} \times a_i d\theta dz \]  
(8)

**B. Equivalent PM Model**

The ESL model significantly reduces the computation time of the Lorentz force; however, the magnetic flux density must be integrated numerically. For real-time applications, it is desired to have the magnetic field solutions in closed form; this can be achieved by modeling the coil as an equivalent permanent magnet (PM) with an effective radius \( a_e \), length \( \ell \), and an effective magnetization vector \( \mathbf{M}_e = M_e \mathbf{e}_z \), the magnetic field solutions of which can then be presented in closed form using distributed multiple dipoles (DMP) [6]. The effective magnetization vector \( \mathbf{M}_e \) is determined to satisfy the following condition:

\[ B_{\text{PI}} = \mathbf{B}(0, 0, \ell/2) \times e_z \]  
(9)

where \( \mathbf{B}(0, 0, \ell/2) \) is the magnetic field of the original coil. For a field that is continuous \((\mathbf{\nabla} \times \mathbf{B} = 0)\) and irrotational \((\mathbf{\nabla} \cdot \mathbf{B} = 0)\), a scalar magnetic potential \( \Phi \) can be defined such that the magnetic intensity \( \mathbf{H} = -\nabla \Phi \). The general expression [13] of \( \Phi_d \) from a magnetic surface charge at \( \mathbf{R}(x', y', z') \) to the field point \( \mathbf{R}(x, y, z) \) is given in (10):

\[ \Phi_d = \frac{1}{4\pi} \int \mathbf{\nabla} \mathbf{M} dV + \frac{1}{4\pi} \int \mathbf{M} \cdot \mathbf{n} dS \]  
(10)

The corresponding magnetic flux density can be derived from the constitutive relation \( \mathbf{B} = \mu_0 \mathbf{H} \). For a cylindrical PM,

\[ B_{\text{PI}} = 0.5 \mu_0 M_o \left[ 1 + \left( \frac{a_e}{\ell} \right)^2 \right]^{1/2} \]  
(11)

The magnitude of the effective magnetization vector can then be obtained by equating (9) and (11):

\[ \mu_0 M_e = 2\sqrt{1 + \left( \frac{a_e}{\ell} \right)^2} \mathbf{B}(0, 0, \ell/2) \times e_z \]  
(12)

where \( \mathbf{B}(0, 0, \ell/2) \) can be computed from (8).

A closed-form solution approximating the magnetic field of the coil is given by the DMP model that has \( k \) circular loops with strengths \( \mathbf{M}_j / \ell \) parallel to \( \mathbf{M}_z \):

\[ B_{\text{DMP}} = \frac{1}{4\pi} \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left( \frac{a_{\text{uni}}}{R_{ij}} - \frac{a_{\text{uni}}}{R_{ij}'} \right) \text{ and } n_j = \begin{cases} 1 & \text{if } j = 0 \\ n & \text{if } j \neq 0 \end{cases} \]  
(13)

where

\[ a_{\text{uni}} = \frac{1}{R_{ij}^2} \left[ (x - \bar{a}_j \cos \theta)^2 + (y - \bar{a}_j \sin \theta)^2 + (z + \ell/2)^2 \right]^{1/2}; \]

\( \bar{a}_j \) is the distance between the source/ sink of the dipole; and \( a_{\text{uni}} \) is the radius of the \( j^{th} \) loop. A general method for finding an optimized set of distributed dipole parameters \( (\bar{a}_j, \ell, k, n, \delta \text{ and } m_j) \) to characterize a PM can be found in [6].

**III. SIMULATIONS AND MODEL VALIDATION**

We present two sets of simulation results. The 1st set illustrates the ESL model, and examines its effects of coil geometries on the magnetic flux density and the Lorentz force. The 2nd simulation validates the equivalent models and compares the computed forces against published data.

**A. ESL model and Effects of Coil Geometry**

Table 1 compares three different coil geometries and their effects on the magnetic flux density for the same wire volume of 5.41 cm³ and length \( \ell = 25.4 \text{ mm} \).

**Table 1: Effects of the ML coil geometry on ESL model**

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Current density</th>
<th>% Error at ( z = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_e/\ell )</td>
<td>( \ell = 1 \text{ mm} )</td>
<td>( \ell = 1 \text{ mm} )</td>
</tr>
<tr>
<td>0.1</td>
<td>0.3256</td>
<td>0.582</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3740</td>
<td>0.766</td>
</tr>
<tr>
<td>0.9</td>
<td>0.7437</td>
<td>0.951</td>
</tr>
</tbody>
</table>

Volume=5.41 cm³ (0.33 in³); \( \ell = 25.4 \text{ mm} \) (1 in); 29AWG wire with 4A current.

**Fig. 3** (\( \ell = 1 \text{ mm} \))

3(a) Original ML (\( \ell = 1 \text{ mm} \))
3(b) \( a_e/\ell = 0.1 \) (\( \ell = 1 \text{ mm} \))
3(c) \( a_e/\ell = 0.5 \) (\( \ell = 1 \text{ mm} \))
3(d) \( a_e/\ell = 0.9 \) (\( \ell = 1 \text{ mm} \))

**Fig. 4** (\( \ell = 1 \text{ mm} \))

4(a) Original ML (\( \ell = 1 \text{ mm} \))
4(b) \( a_e/\ell = 0.1 \) (\( \ell = 1 \text{ mm} \))
4(c) \( a_e/\ell = 0.5 \) (\( \ell = 1 \text{ mm} \))
4(d) \( a_e/\ell = 0.9 \) (\( \ell = 1 \text{ mm} \))

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With the current of 4 Amperes, the effective radius \( a_r \) and current density \( J_r \) in Table 1 were found numerically using the optimization toolbox in MATLAB. Since the magnetic flux density of the ESL model is singular at the surface \( z = l/2 \), we compare \( B_{ax}(y, l/2 + \varepsilon) \) at \( \varepsilon = 1\mu m \) and 1 mm in Figs. 3 and 4 respectively for three different ratios \( a_l/a_r \leq 0.1, 0.5 \) and 0.9. The errors of the ESL models are summarized in Table 1 where the % Error is defined as

\[
\text{%Error} = \frac{\int_0^l |B_{ax}(y, z) - B_{ex}(y, z)| dy}{\int_0^l B_{ex}(y, z) dy} \times 100 \%
\]

Comparison of results in Fig. 3, Fig. 4 and Table 1 shows that the ESL model well approximates magnetic fields of coils with \( a_l/a_r \geq 0.5 \). As expected, discrepancies between the ML and ESL models occurs primarily at the surface \( z = l/2 \), particularly for coils with a very small \( a_l/a_r \) ratio. This implies that one or more additional wire layers may be needed to improve the approximation. For the same volume of wires and coil length, thin coils \( (a_l/a_r = 1) \) tend to have a more uniform but lower magnetic flux density along the centroidal axis than that of the thick coil \( (a_l/a_r << 1) \).

Figure 5(a) shows the setup used to compare the torque computed with the 3D field of the ESL model \( (8) \) against that based on the 3D field of the original ML coil \( (7) \), where the cylindrical PM is rotated in the \( xz \) plane towards the ML coil. The parameters of the PM and coil are given in Table 1. Computation with the ESL model requires only 5% of the computation time with the original ML coil in MATLAB. As shown in Fig. 5(b), the two computed torque are in excellent agreement. The parameters of the PM and coil are given in Fig. 5(a).

### B. ESL Model Validation with Force Computation

To validate the equivalent models, we model the setup in Figs. 6(a) and 7(b) respectively against published experimental data and numerical results computed using mesh-less method (MLM) [14]. As shown in Fig. 7, both the original ML coil and equivalent SL model agree very closely with each other and with the MLM. Maximum differences from the experimental data, \( 100 \times | F_{ext} - F_{exp} | \), are within 10% as shown in Table 3.

![Image of experimental setup and parameters](image)

![Image of parameters used in simulation](image)

**Fig. 5 Effect of equivalent models**

![Image of experimental setup and parameters](image)

**Fig. 6 Experimental setup and parameters**

**Table 2 Parameters of the equivalent SL models**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Large</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM (DMP) ( k=2; n=6 )</td>
<td>( \delta )</td>
<td>( m_i (\mu T/m^3) )</td>
</tr>
<tr>
<td></td>
<td>0.3140</td>
<td>0.3122</td>
</tr>
<tr>
<td>ESL of ( n_j=1, 2, 3 )</td>
<td>( J_{dl} (A/m) )</td>
<td>( 2.750 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

**Table 3: Maximum difference from published experimental data**

<table>
<thead>
<tr>
<th>Multilayer (%)</th>
<th>Single layer (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangential / Axial force</td>
<td>5.79 / 8.21</td>
</tr>
</tbody>
</table>

![Image of tangential forces (large)](image)

**Fig. 7 Comparison between computed and experimental results**
IV. ILLUSTRATIVE APPLICATIONS

Two examples have been simulated. Example 1 investigates the effect of pole-shapes on the magnetic torque using DMP of the coils. Example 2 illustrates the B-fields to determine the orientation of a spherical motor.

Example 1: Effect of pole-shape and design configuration

Pole-shape designs have significant influences on the performance of an EM actuator. This example analyzes the effects of two pole-shapes on the torque of a spherical motor: Design A [16] consists of 2 rows of 8 cylindrical PM’s of \( \gamma = 1 \).

Design B [17] uses 8 assemblies of 5 cylindrical PM’s.

We focus on comparing net magnetic torques per unit magnet-volume for a given rotor radius and under the same influence of the stator coils in Fig 10. Detailed geometries of the PM pole-shapes are compared in Table 4 and Fig. 8.

Table 4 Parameters used in simulation

<table>
<thead>
<tr>
<th>Common Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor radius, mm</td>
</tr>
<tr>
<td>( r_1 \times 37.5 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PM Pole Designs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
</tr>
<tr>
<td>1.27</td>
</tr>
<tr>
<td>1.27</td>
</tr>
</tbody>
</table>

Simulated magnetic flux and potential lines are compared in Fig. 9; the potential and flux lines are orthogonal. Figure 9(a), or the left column, compares the magnetic fields of the PM only. Unlike Design B where only one (row of) PM is used, a significant portion of the flux lines in Design A forms a closed path between two PM’s. Once the magnetic field of the PM’s is found the force acting on the current-carrying loops can be calculated using the Lorentz force equation. Figure 10 compares the torque per unit volume of the two designs, which uses the magnetic field given in Fig. 9(a). In calculating the torques, ±1A current profiles in Fig. 10(a) are given to the coils such that a positive torque in +y-direction is generated.

Design A Design B Normalized 2D Bz (PM: Design A)

Fig. 9 Magnetic fields (Orange line: potential; blue lines: magnetic flux)

Top: Design A; bottom: Design B

Fig. 10 Comparison of torque/volume

Example 2: Shaft Inclination Sensor

Figure 11 illustrates the use of DMP models [6] to determine the shaft inclination of a spherical motor by measuring the B-field of the cylindrical PM.

Sensor location:  
S1 = (a/2,0,b);  
S2 = (-a/2,0,b);  
S3 = (0,a/2,b); and  
S4 = (0,-a/2,b)  
where a = 89mm (3.5in); and b = 81mm (3.2in)

Fig. 11 Schematics Illustrating Inclination measurement
The magnetic field of the PM at the $m^{th}$ sensor located $S_m$ can be calculated from (15):

$$B = \frac{\mu_0}{4\pi} \sum_{m} \sum_{j} \left( \frac{S_m-(L+p_{ji})}{|S_m-(L+p_{ji})|} \frac{S_m-(L+p_{ji})}{|S_m-(L+p_{ji})|} \right)$$

(15)

where the distance vector $L$ denotes the location of the PM in the reference XYZ frame

$$L = L[\cos \alpha \cos \beta \cos \alpha \sin \beta \sin \beta]^T;$$

and the dipoles are known with respect to the coordinate frame $xyz$ of the PM:

$$p_{ji} = [\pi_j \cos \theta \pi_j \sin \theta \pm \pi/2]^T$$

(15b)

Equation (15) provides a means to determine the unknown orientation $q(\alpha, \beta)$ from the magnetic flux density $B$ using Hall-effect sensors.

As an illustration, we consider a two-sensor-pairs array on the plane $Z=L$ such that the magnetic flux densities $B_X$ and $B_Y$ along the $X$ and $Y$ directions can be measured by the sensor pair $S_1$ and $S_2$ and the sensor pair $S_3$ and $S_4$ respectively. Figure 12(a) simulates the $B$ fields measured by one of the sensors. Once the magnetic flux density $B$ is known, the incremental change in orientation for real time ($\dot{q} = q_{i+1} - q_i$, where the subscripts denote the time steps) can be computed from the linear equation (16):

$$[A] \dot{q} = b$$

(16)

where

$$b = [B(q_{i+1}) - B(q_i)] \in \mathbb{R}^{4	imes1}$$

(16a)

and

$$[A] = \begin{bmatrix} \frac{\partial B}{\partial \alpha_{i+1}} & \frac{\partial B}{\partial \beta_{i+1}} \\ \frac{\partial B}{\partial \alpha_{i}} & \frac{\partial B}{\partial \beta_{i}} \end{bmatrix} \in \mathbb{R}^{4	imes2}$$

(16b)

Figure 12(b) compares the estimated angles based on the incremental motion (16) against the exact solutions. In this simulation, the inclined shaft rotates about the $Z$-axis at 167 rpm and the sensor updates the measurements four times per revolution.

V. CONCLUSIONS

Two different equivalent models for calculating the magnetic field and forces due to multilayer (ML) voice coils have been presented and validated; namely, an equivalent single layer (ESL) model, and an equivalent PM model. We have investigated the effects of coil geometry on the modeling errors, and also compared the Lorentz forces computed based on both the fields of the original ML coil and the ESL model. The comparisons are in excellent agreement; more importantly, computation with the ESL model requires only 5% of the computation time with the original ML coil in MATLAB. Through two illustrative applications, we demonstrate the use of the equivalent models for pole-shape design and field-based orientation sensing. Unlike numerical solutions such as FEM, the magnetic field solutions obtained using the equivalent PM model are in closed form and thus well suited for real-time computational applications.

REFERENCES


