DEVELOPMENT OF A VISION BASED ORIENTATION SENSOR

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ABSTRACT

The development of a vision based orientation sensor for the measurement of the orientation of a spherical body is presented. A specially designed grid pattern used to encode absolute orientation information onto a spherical surface in terms of latitude and longitude angles is discussed. A novel algorithm for the analysis of images of this grid pattern based on tracking along the grid lines is also developed that is designed to overcome limitations of other traditional image analysis techniques in terms of speed and accuracy. A procedure for recovering the absolute orientation of a spherical surface encoded with the special grid pattern using the line tracking algorithm, camera calibration, and Horn's (1987) technique is also presented. Finally, experimental results of this procedure applied to a sequence of images are presented and discussed.

1. INTRODUCTION

Many single degree-of-freedom feedback control systems use relative measurement systems that may give satisfactory performance for linear systems. However, in many nonlinear systems the control forces applied to the system depend on the absolute position of the system because of its nonlinear nature, and a measurement system capable of absolute measurement is required.

For multi-degree-of-freedom ball-joint-like devices such as robotic wrist actuators and joysticks with three concentric axes of rotation, three consecutive-rotary optical encoders are often used to perform the orientation measurement. Such measurement configurations have several disadvantages. First, to measure the spin of these devices additional guide structures are often needed, which add mass and friction to the overall system. The optical encoders require contact with the shafts being measured, which add friction. Relative optical encoders would also require some special calibration procedure. Due to the drawbacks of the three single-axisencoder configuration, an improved orientation measurement system is desired that will provide non-contact absolute orientation measurement of a spherical object. Such a measurement system would eliminate the need for the guide structures and reduce the friction on the system. Lee (1993) has proposed a vision-based approach to solve this measurement problem that uses a special grid pattern to encode orientation information onto a spherical surface.

For this vision based measurement system, grid pattern images must be carefully processed to extract the desired information. The fundamental issues related to processing the grid pattern images involve determination of the line intersections as accurately as possible and maintaining the proper sequence of the intersection points. To be useful for a real-time vision based measurement system, an algorithm that carries out this task must meet several requirements. First, it must be as fast and efficient as possible. It must also be very robust to changes in the conditions of the image of the grid pattern. Finally, it must be able to locate the line intersection information as accurately in the image as possible to give highly accurate measurement results. In the best case, this means that the line intersections should be located with sub pixel accuracy.

In the area of machine vision, there are widely used techniques for the location of features such as lines. One of the most well known is the Hough transform (Leavers 1993), which transforms significant feature points in the image to curves in a phase space parameterized such that curves corresponding to points that could occupy a straight line will overlap. The phase space can be analyzed to determine where a large concentration of curves overlap, which correspond to lines in the original image. There are several reasons why the Hough transform is not suitable for real-time analysis of the grid pattern image. First, to determine the feature points used by the Hough transform, the entire region of interest of the image must be processed, which is a time consuming global image processing step. To achieve greater accuracy in locating line intersections, the resolution of the phase space used by the algorithm must be increased, which also increases the processing time. Finally, the Hough transform can only identify lines in the image; determination of line intersections would require an additional step, and finding the proper sequence of intersections along a line would require additional processing involving a time consuming exhaustive search. Therefore, the Hough transform does not seem to be a suitable candidate for the analysis of the grid pattern images.

A task similar to the grid pattern image analysis is found in the analysis of the blood vessels in coronary angiogram images and eye-fundus blood vessels (Kurokawa 1998, Sen 1999). In the analysis of these images that contain the branching structure of blood vessels, algorithms are used to track along the blood vessels and record the variation in their size for diagnosis of various diseases. These algorithms must be able to deal with the branching of the blood vessels, which is very similar to the intersection point determination for the grid pattern image analysis described above. Tracking along a particular line and locating the intersection points found along it also appears to be a more natural approach to the grid pattern analysis problem described above, because such an approach is an efficient technique that emulates how a human operator would approach such a problem. The task of analyzing the grid pattern image using such a line tracking technique requires implementation of an efficient algorithm that will meet the requirements stated above.

In this paper, the background related to the vision based absolute orientation measurement system and a novel technique for analyzing the grid pattern images will be presented. The information obtained from this novel image analysis technique will be used with a closed-form absolute orientation technique developed by Horn (1987) to recover the absolute orientation of the spherical body. Finally, experimental results will be presented and discussed.

2. SYSTEM OVERVIEW

2.1 Grid Pattern

In order to measure the absolute orientation or position of an object, there must be some frame of reference. For a vision-based measurement system, a frame of reference can be encoded onto the object to be measured as a specially designed pattern, and the camera system can then measure the movement of the object by analysis of the pattern.

Figure 1 shows an image of spherical shell positioner apparatus designed and built by Klement (1997) that is capable of drawing specially designed patterns of latitude and longitude lines onto a spherical surface. This apparatus consists of a spherical shell with a grid pattern drawn on it mounted onto a central shaft that is actuated by three stepper motors. Figure 2 shows an image of the grid pattern on the spherical shell taken with a CCD camera. This pattern consists of a number of latitude and longitude lines that are not uniformly spaced on the surface of the sphere. The spacings between the lines are chosen such that for some specified number of spacings, the spacing sequence will be unique. In this way, the absolute orientation information can be encoded into the grid pattern in a way that can be easily analyzed by a vision system, which will define a frame of reference on the spherical shell.



Figure 1. Image of Spherical Shell Positioner for Grid Pattern Drawing



Figure 2. Grid Pattern Image

2.2 Line Tracking

For any particular line in the image the sequence of spacings between some number of lines that intersect the line must be determined. The problem of ascertaining this information from the image can be broken into two important aspects.

- First, there must be an efficient and robust means of locating the intersection points in the grid pattern image.
- Second, the correct sequence of intersections along a particular line must be known.

The line-tracking problem for the grid pattern image can be posed as the following. Given an initial starting point in the image that lies on a grid line and a direction along the line, the line should be tracked and the intersection points with perpendicular lines must be located. To accomplish this task it is necessary to gather some local information about the line being tracked, in order to correctly proceed along the line and accurately locate line intersections. In the blood vessel tracking algorithms mentioned above, some type of cross section perpendicular to the tracking direction is used or a local area of pixels are analyzed in a special way to determine how the tracking algorithm should proceed. One example of such an approach relies on examination of a region defined as some portion of a circle, which emulates the sonar tracking capability of bats.

This technique is illustrated in Figure 3 below. Figure 3(a) shows the case with a straight line with no intersection points. The technique examines the gradient profile along radial lines starting at the current center location and terminating on the circular arc. As indicated by the shaded region, the portion of the circular arc that does not contain any significant edges is identified as being part of the line to be tracked. The location of this portion of the circular arc is used to determine the next center point for the circular arc and the correct direction to proceed. The portion of the circular arc in which edges of the line are found can also be used as a means of keeping the circular arc centered on the line. Figure 3(b) shows the case of a line intersection with three regions of the circular arc where the radial profile will find no line edges. The location of these three regions can be used to determine the directions of the intersecting line and its location to continue tracking along it if desired.



Figure 3. Circular Arc Profile Tracking Method

An algorithm based on this circular arc technique has been implemented for the grid line tracking application. It was found to perform reasonably well in the form previously described, but several enhancements were made to optimize it for use in the grid line tracking application.

- First, instead of using a number of radial gradient profiles that examine the entire area enclosed by the circular arc, the gradient profile along the circular arc can be used to determine essentially the same information. Therefore, the gradient value is examined at significantly fewer pixel locations, which will give an increase in the speed of the algorithm.
- The circular arc shape has also been changed to a boxlike shape consisting of three line segments.

Figure 4 illustrates the box-like profile, which has been found to be more useful in tracking the grid lines than the circular arc profile because the box-like profile matches the shape of the grid lines better than the circular arc. It has also been found it performs better in tracking and locating the intersection points of closely spaced lines because the lengths of the three line segments can be easily adjusted. However, only the radius of the circular arc can be adjusted, which has been found to make it difficult to track along closely spaced lines without missing line intersections.



Figure 4. Box-Like Profile Tracking Method

2.3 Absolute Orientation Determination

The task of determining the absolute orientation of a body with respect to a fixed frame of reference is equivalent to finding the transformation between the coordinates of corresponding points in the fixed frame and another reference frame. The relationship between two sets of coordinates in two different reference frames is given by

$$\mathbf{r}_{sph} = sR\mathbf{r}_{W} + \mathbf{r}_{0} \tag{1}$$

where \mathbf{r}_{sph} represents a vector containing a coordinate in the sphere reference frame, *s* is a scale factor, *R* is a rotation matrix that transforms coordinates from the fixed world frame to the sphere frame, \mathbf{r}_W represents a vector containing a coordinate in the fixed world reference frame, and \mathbf{r}_0 is some translation vector. The desired absolute orientation between the sphere frame and the world frame will be given by the rotation matrix in Equation (1). To determine the rotation matrix *R*, scale factor *s*, and the translation vector \mathbf{r}_0 , all of the corresponding points must be used to find a solution in an optimal sense.

In Equation (1) only two quantities are known which are the coordinates of a particular point in the sphere frame \mathbf{r}_{sph} and the corresponding coordinate of the same point in the fixed world frame \mathbf{r}_{w} . With a sufficient number of corresponding points, Equation (1) can be solved for the unknown parameters. As mentioned previously, the grid pattern is specially designed to encode the absolute orientation on to the spherical surface to define a frame of reference on the sphere. By tracking two perpendicular lines in a grid pattern image as shown in Figure 5, the latitude and longitude angles corresponding to each of the intersection points in the image can be found by matching the spacing between each of the intersection points with the known spacing sequence used to generate the grid pattern. This matching is done using scaling factors found from a normalized matching procedure that relates the pixel values to the spacings in the known spacing sequence. The sequence matching has been found to yield matching results with an average deviation from the known sequence of 1% to 5%.



Figure 5. Image with 2 Perpendicular lines Tracked

Figure 6. Image with All Lines Tracked

With the latitude and longitude angles corresponding to each of the intersection points in the image and the known radius of the sphere, the coordinates of the points in the reference frame attached to the sphere can be found using the definitions for spherical coordinates to give the \mathbf{r}_{sph} values for the points in the image. To determine the corresponding coordinates of the points in the image in the fixed world frame to find the \mathbf{r}_{W} values, a machine vision technique known as camera calibration must be used. Camera calibration allows the orientation of a camera with respect to a fixed coordinate frame defined by a calibration object and other parameters associated with the camera and lens system to be determined using a least squares technique. The point correspondences necessary for the camera calibration can be easily found using the line tracking procedure to find as many intersection points in the grid pattern image as possible, which is equivalent to using the grid pattern on the spherical shell as a calibration object. Figure 6 shows a grid pattern image with 146 intersection points located as indicated by white pixels. Using the \mathbf{r}_{sph} value for each point and its pixel coordinate in the image, a Direct Linear Transformation camera calibration procedure has been used to determine the camera parameters (Abdel-Aziz, 1971).

The 3D points in the world frame \mathbf{r}_W can be found using a stereo technique with two calibrated cameras, and in the case of the spherical shell, a single camera can be used with the constraint that the points must lie on a sphere. With the set of corresponding 3D points in the sphere frame \mathbf{r}_{sph} and in the world frame \mathbf{r}_W , the unknown parameters in Equation (1) can be found using a technique proposed by

Equation (1) can be found using a technique proposed by Horn (1987) that uses unit quaternions to represent the rotation transformation. This technique gives a closed-form solution to the absolute orientation between the two frames, which is very desirable for a real-time implementation. Horn's technique for determination of the absolute orientation has been implemented and tested with data from several grid pattern images as will be presented in §4.

3. Box-Shape Profile Line Tracking Algorithm

The fundamental line-tracking algorithm utilizing the box-shaped profiles consists of four basic steps, which are listed here and described below.

- 1. Determine starting point and direction for tracking in image. This can be done by a human operator or by an automated technique.
- 2. With the starting point and direction information, construct the box-like profile and extract the gradient value along the profile.
- 3. Process the gradient profile to determine how to proceed by locating dark features that correspond to the lines to be tracked. The possibilities are the following:
 - a. No dark features stop or try to proceed with previous information
 - b. One dark feature tracking along a single line, proceed forward to the midpoint found
 - c. Two dark features likely near an intersection point, make small adjustment to box shape and try again to find intersection
 - d. Three dark features intersection point found, use edge information to find center point corresponding

to line intersection and return the angles and midpoints between edges for front, left, and right line segments so tracking may continue along them

- e. More than three dark features stop or try adjusting box shape parameters due to possibility that the box shape is too large
- 4. Use result of step 3 and go back to step 2 and continue iteration to perform the tracking process until some desired number of intersection points are found or the image boundary is reached.

The primary results of this algorithm are the intersection points along a particular line in the proper sequence. Other important information may also be recorded such as the direction angles corresponding to the intersection points, which can be used to track along the perpendicular lines. Each of the steps in the algorithm will now be discussed.

3.1 Starting Point Determination

For a real-time line-tracking algorithm, there must be an automated technique for determination of the starting point and correct tracking direction for the algorithm. To minimize distortions due to the imaging system, it is also desirable to start the tracking as near to the center of the image as possible. For this reason, the intersection point in the image closest to the center of the camera field of view should be determined. This region was binarized using an optimal thresholding technique. The Zhang-Suen (1984) thinning algorithm was then used to thin the resulting binary image to give the thinned result that appears in Figure 7. This thinned region is traversed to locate pixels with connectivity greater than two, which strongly indicates that they are on or near an intersection point. The pixel locations near each other are then clustered together to give an average location, which was used to determine the cluster closest to the center pixel in the image. This closest value was taken to be the central intersection point, and a square was traversed around this point to determine the four intersection points that were taken to correspond to the two intersecting lines as shown in Figure 7 below. Finally, these four points were used to find the associated direction angles for each of the two lines.



Figure 7. Image with Thinned region and central intersection point located

This automated central intersection point location technique based on the analysis of a thinned image has been found to perform very well in a number of images with differing translations and rotations. It is also significantly faster and more robust than an approach based on the Hough transform that has been investigated.

3.2 – Gradient Profile along Box-Like Shape

The next step is to find the gradient profile along the box-like shape at some location in the image. As shown in Figure 4 above, the box-like shape consists of three line segments with two parallel segments and another perpendicular one that joins the two. The lengths of the front line segment and the two side segments are defined based on the width of the grid lines in the image. With these lengths, the desired location in the image, and the angle, the locations of the box shape in the image can be determined in terms of the endpoints of each of the three line segments. Using these four points, the Bresenham (1965) line algorithm has been implemented to efficiently locate the pixels that lie on each of the line segments. Figure 8 shows a close up view of a grid image with several box shaped profiles superimposed on the image. As the line algorithm proceeds along each line segment, the gradient value at each pixel is also calculated and stored. The result is a gradient profile along the box shape. It is also essential to record the coordinates of each of the pixels corresponding to values in the gradient profile, so that the locations of the edges along the gradient profile can be determined.

3.3 Process Gradient Profile

With the gradient profile along the box shape, the intensity profile, and the coordinates of the pixels that make up the box shape, dark features in the intensity profile that correspond to the grid lines in the image can be found. Efficient and robust location of these dark features is essential because it gives the information necessary to perform accurate line tracking and intersection point location.





Figure 8. Close up View of Grid Image with Box Shaped Profiles





Figure 9. Single Line Case [A]



Figure 11. Line Intersection Case [C]

As stated above, the algorithm must analyze the intensity profile to determine the number of dark features present in the profile, so that the next iteration will proceed correctly. Figure 8 shows a close-up view of a grid image with several box shapes superimposed on it. Figures 7-9 show plots of typical examples of the gradient and intensity profiles for cases with single lines, line intersections, and indeterminate situations. In each of the plots, the normalized gradient profile is the solid line, and the normalized intensity profile is the dashed line.

Figure 9 shows the normalized gradient and intensity profiles for a case with only a single line. This determination is made because there are two peaks present in the gradient profile that correspond to two edges of a single dark line. Such a result would be expected from a box shape like that denoted by A in the image in Figure 8. It is important to note that the location of the peaks in the gradient profile is a crucial part of the line tracking algorithm because this information is used to determine which direction to proceed with the tracking and where the line intersection points are located. For this reason, the peaks in the gradient profiles are located by taking the weighted average of the gradient values and their pixel coordinates in order to locate the peaks in the gradient profile with sub pixel accuracy. It is also important to note that all of the gradient profiles are normalized by dividing the magnitude of the gradient at a particular pixel found from the Sobel operator by the intensity value of the pixel. This has been found to give normalized gradient profiles in which significant peaks will usually have values that exceed one due to the normalization procedure. With this approach, the value of one can be used as a threshold for locating the peaks in the gradient profile, rather than choosing an optimal value of the threshold for the gradient profile. Such an approach would also be expected to perform well with images in which the contrast may not be optimal.

Figure 10 shows a normalized gradient profile for an indeterminate case that would be typical of a box shaped profile near an intersection point like that denoted by B in Figure 8. This type of profile is indeterminate because it contains a set of four peaks that could be used to identify two different dark features. The algorithm must be able to determine which of these possibilities should be used to achieve accurate and robust tracking. This can be done by recording the segment of the box shape on which each of the peaks in the gradient profile is found to allow the algorithm to distinguish between cases when the wrong dark featured would be identified, which could result in stepping over an intersection point.

Figure 11 shows a normalized gradient profile for a box shape that lies at a line intersection, which would be typical of a box shape like that denoted by C in the image in Figure 8. The gradient profile contains six peaks that correspond to the edges of the three dark features that the box shape passes through. The pairs of peaks that correspond to each of these dark features can be identified with the use of the intensity profile, and the midpoints between the edges of these features can be found. With these midpoints, the intersection point can be determined as illustrated in Figure 12. First, the dark feature midpoints on the two parallel segments of the box shape are used to define a line. The other dark feature midpoint on the segment perpendicular to the other two is used to determine the point on the line closest to it. This point is taken to be the intersection point between the two lines. This technique has been found to perform very well for location of the intersection points of the grid pattern images with sub pixel accuracy.



Figure 12. Line Intersection Point Determination

Obviously, the three cases just mentioned are not the only possibilities that will occur when tracking lines in a grid pattern image. For optimal operation of the algorithm, it is very advantageous that only cases with a single dark feature or three dark features be found, which will result in the most efficient line tracking because the box shape will either be on a single line or at an intersection point. Therefore, great care must be taken to analyze cases that do not fit these criteria to determine if they can be made to fit one of these cases to optimize the performance of the algorithm. As mentioned previously, an example of this is found in the indeterminate case in Figure 10, in which the correct dark feature must be identified. The other significant example of this type of case would be if only two dark features could be identified, which would indicate that the box shape was near an intersection point. In such a case, the gradient profile is analyzed to give only a single dark feature so that the box shaped profile will only be adjusted by a small amount, so it will not move over an intersection point. In an analogous way, if more that three dark features are identified, the algorithm will try to determine which ones should be used to define the line intersection point.

The result of the processing of the gradient profile will be a set of points which lie on the midpoints of each of the dark features found along the gradient profile, and the intersection point for the case of three dark features found. These points are also used to determine the angle between them and the current box shape location to update the direction angle of the box shape. For the case of a single point, this point is used as the new box shape location as it moves along the line. For the case of three points at a line intersection, the three points and associated direction angles may be used to track the grid lines that are perpendicular to the one being tracked.

4. EXPERIMENTAL RESULTS

An experimental test has been performed using an apparatus that appears in Figure 1, which is capable of moving the spherical shell to a desired orientation. For testing purposes, a trajectory was performed that consists of the following movements. First, the shell was made to nutate from an initial home position by 10 degrees to a starting position. Then it was commanded to nutate in the opposite direction by 20 degrees, then to precess by 90 degrees while spinning by 90 degrees, then to nutate by 20 degrees, and

finally to precess by 90 degrees while spinning by 90 degrees back to the starting position. A sequence of six images was taken as the shell carried out this trajectory.

Two perpendicular line intersection sequences similar to those in Figure 5 were found in each image, and they were matched to find the latitude and longitude angles corresponding to each point, which was used to calculate their 3D coordinates in the sphere frame. This information was used with the previously determined camera calibration parameters to find the 3D coordinates of each of these points in the world coordinate frame. With this set of corresponding points, Horn's technique was used to determine the absolute orientation at each step in the trajectory.

Table 1 shows the results from the analysis of the image sequence. For each image the axis-angle representation of the rotation matrix representing the absolute orientation from the initial position as determined from the information from the image is shown along with the axis-angle representation of the corresponding actual commanded motion (Sciavicco 1996).

Table 1. Experimental results From Image Sequence

Image	Actual		Calculated	
	Angle (deg)	Axis	Angle (deg)	Axis
#1	0	[1]	0	Arbitrary
		0		
		o		
#2	10	[1]	9.8027	[0.9959]
		0		0.0297
		o		0.0928
#3	10	[-1]	8.4868	[-0.9798]
		0		-0.1504
				0.1199
#4	10	[0]	10.6731	[-0.0086]
		-1		-0.9560
				0.2924
#5	10	[0]	11.2457	[0.0059]
		1		0.9581
		[0]		0.2861
#6	10	[1]	9.8027	[0.9970]
		0		0.0250
		[0]		0.0919

The results found for the absolute orientation from the grid images analysis are in close agreement with the actual orientation of the spherical shell at each step of the trajectory. Discrepancies are primarily due to nonlinearities that result from the use of a cardan joint in the spherical shell positioning system. These discrepancies are noticeable in the small nonzero terms in the axis vector. The stepper motors used in this positioning system are also operated in open loop mode without the benefit of encoders for measurement of the motor angles. To overcome these limitations, a simulation of the positioning of the spherical shell has been implemented using OpenGL in a Visual C++ application, which allows synthetic images of the grid pattern to be obtained with a known orientation. This simulation environment not only

allows for arbitrary orientation of the spherical shell specified by three orientation angles, but also is easily able to specify arbitrary translations of the spherical shell, which simulates a true six degree-of-freedom positioning system. In practice, such a six degree-of-freedom positioning system may require custom design and cost at least \$100,000, and there will be inherent limitations in the range of motion of such a system. Therefore, the use of the OpenGL environment for modeling of the imaging of the grid pattern on the spherical shell has a number of advantages over a physical setup in terms of flexibility and cost. This simulation environment will be used as a means of verifying the performance of the vision based measurement system in the ideal case, and it will be used as a means of testing different grid configurations before they are actually implemented.

5. CONCLUSIONS AND FUTURE WORK

In this paper an algorithm for the analysis of the intersection points in grid pattern images using a linetracking approach for use in a real-time vision based orientation measurement system has been presented. This algorithm has several advantages in terms of speed and accuracy as compared with traditional techniques such as the Hough transform. A special grid pattern that encodes the absolute orientation information in the form of latitude and longitude angles has also been described. This grid pattern is also used as a calibration object for a camera calibration procedure necessary to locate the intersection points in the grid pattern image in a fixed world coordinate frame. The set of corresponding points found using the line tracking algorithm and the calibrated camera have been used with Horn's closed-form absolute orientation recovery technique using unit quaternions to find the absolute orientation of the spherical shell with respect to a fixed frame of reference. Finally, results of the application of this procedure to a sequence of images taken at known orientations of the spherical shell has been presented and discussed.

In the future, the vision based orientation measurement procedure presented in this paper will be further tested using synthetic images generated in an OpenGL modeling environment to demonstrate and validate its capability to recover the orientation and translation of a spherical object. This will give a true six degree-of-freedom vision based measurement system. Due to limitations of the current grid pattern design in terms of range of motion, other pattern designs will be tested and evaluated using the OpenGL environment before being implemented. These patterns would not necessarily have to be on a spherical surface because the line-tracking algorithm will work for any similar grid pattern on other types of regular surfaces. Finally, the vision based measurement procedure will be implemented with a high speed CMOS camera system capable of frame rates up to one thousand frames per second to give a sampling time of one millisecond suitable for use in a feedback control system.

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